The Slutsky Effect

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The *Slutsky effect*, as understood by many economists, is described in the following quote from Barnett (2006):

> If the variables that were taken to represent business cycles were moving averages of past determining quantities that were not serially correlated either real-world moving averages or artificially generated moving averages then the variables of interest would become serially correlated, and this process would produce a periodicity approaching that of sine waves.

To demonstrate this effect we take a sequence of independent identically distributed random variables with mean μ and variance σ^2, compute a moving average, and look for oscillatory behaviour. (The example that follows is similar to Royama p. 131.)

Suppose that u_i is a sequence of independent identically distributed random variables . Here we generate 1000 samples, shown in Figure 1.

```
set_random_seed(0)

def avg(L):
    return sum(L)/(1.0*len(L))

def variance(L):
    mu = avg(L)
    return avg([(x - mu)**2 for x in L])
```

```
nr_samples = 1000
N = 10 # u_i is in [0, 1, ..., N - 1]

U = [randrange(N) for _ in range(nr_samples)]

U_mu = avg(range(N))
U_var = variance(range(N))
```

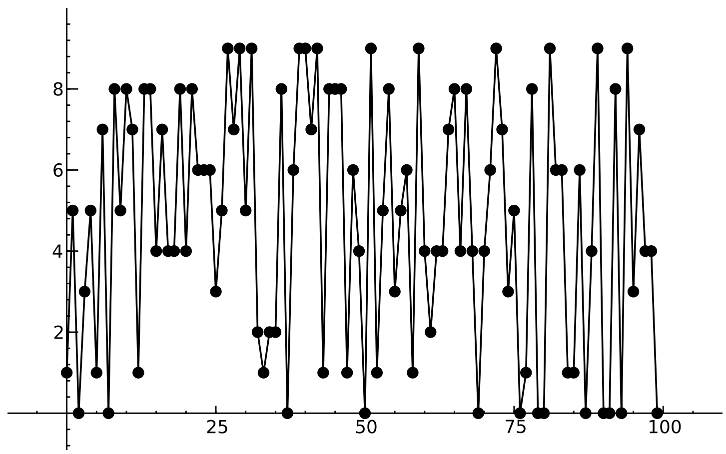

Figure 1: First 100 points from sequence u_i.

There is no trend nor systematic behaviour of the series. We can check this by calculating the autocorrelation $R(t)$:

$$R(t) = \frac{\mathbb{E}((u_i - \mu)(u_{i+t} - \mu))}{\sigma^2}.$$

The autocorrelation ranges between -1 and 1, meaning perfectly anti-correlated and perfectly correlated, respectively.

```
def autocorrelation(L, t):
    """
    Biased estimator of the autocorrelation.
    """
```

```
    mu = avg(L)

    return avg([(L[i] - mu)*(L[i+t] - mu)
        for i in range(len(L) - t)])/variance(L)

t_max = 20

U_ac = [autocorrelation(U, t) for t in range(t_max)]
```

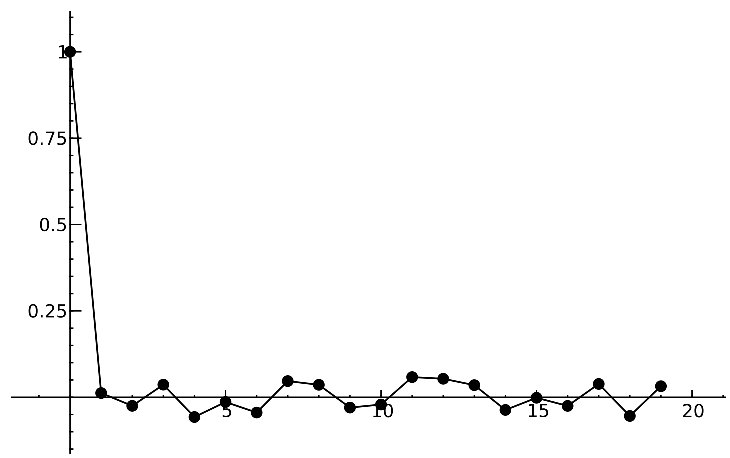

Figure 2: Autocorrelation of the sequence u_i.

This is as expected. In Figure 2 We see that $R(0) = 1$ because any sample is autocorrelated with itself, and $R(t)$ is around 0 for all $t > 0$ since the samples are independent. So what happens if we take a moving average of the series? Define a new random number w_t by

$$w_t = \frac{1}{k} \sum_{i=1}^{k} u_{t-i+1}.$$

Below we compute this new series for $k = 10$ and plot the autocorrelation.

```
moving_avg_len = 10
```

```
W = [(1.0/moving_avg_len)*sum(U[i:i+moving_avg_len])
    for i in range(len(U) - moving_avg_len)]

p = list_plot(W, plotjoined = True, marker = 'o', figsize = 5,
    rgbcolor = 'black')
p.save(filename = "W_plot.pdf", ymin = 0)
```

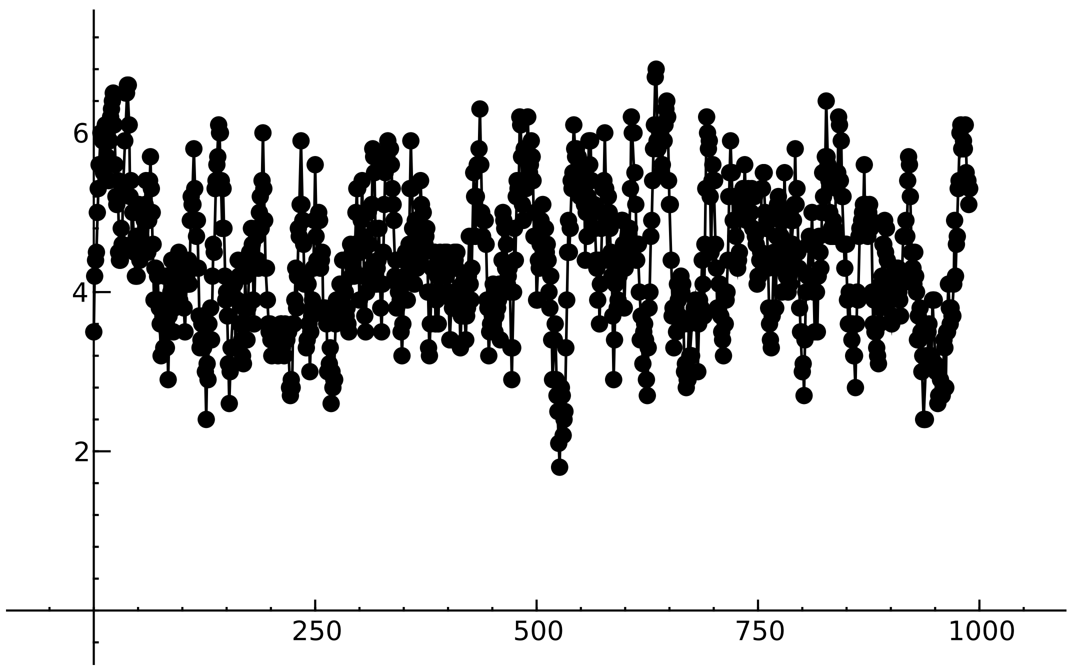

Figure 3: Sequence w_i.

Figure 3 shows oscillatory motion. This is because two points that are j points apart share $k - j$ points of the original series (if $j < k$) and 0 otherwise. We see this in the autocorrelation, calculated as follows, and shown in Figure 4.

```
W_ac = [autocorrelation(W, t) for t in range(t_max)]
```

If we repeat the taking of a moving average then we see (Figure 5) what appears to be very regular behaviour:

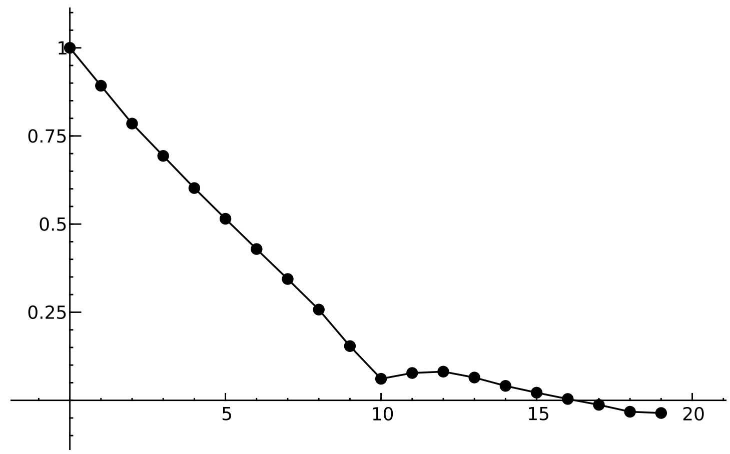

Figure 4: Autocorrelation of the sequence w_i.

```
W1 = [(1.0/moving_avg_len)*sum(W[i:i+moving_avg_len])
    for i in range(len(W) - moving_avg_len)]

W2 = [(1.0/moving_avg_len)*sum(W1[i:i+moving_avg_len])
    for i in range(len(W1) - moving_avg_len)]

W3 = [(1.0/moving_avg_len)*sum(W2[i:i+moving_avg_len])
    for i in range(len(W2) - moving_avg_len)]

W3_plot = list_plot(W3, plotjoined = True, marker = 'o',
    figsize = 5, rgbcolor = 'black')
W3_plot.save("W3_plot.pdf", ymin = 0)
```

It is interesting (but not surprising?) that the points in Figure 5 stay near the mean of the original, 4.39600000000000.

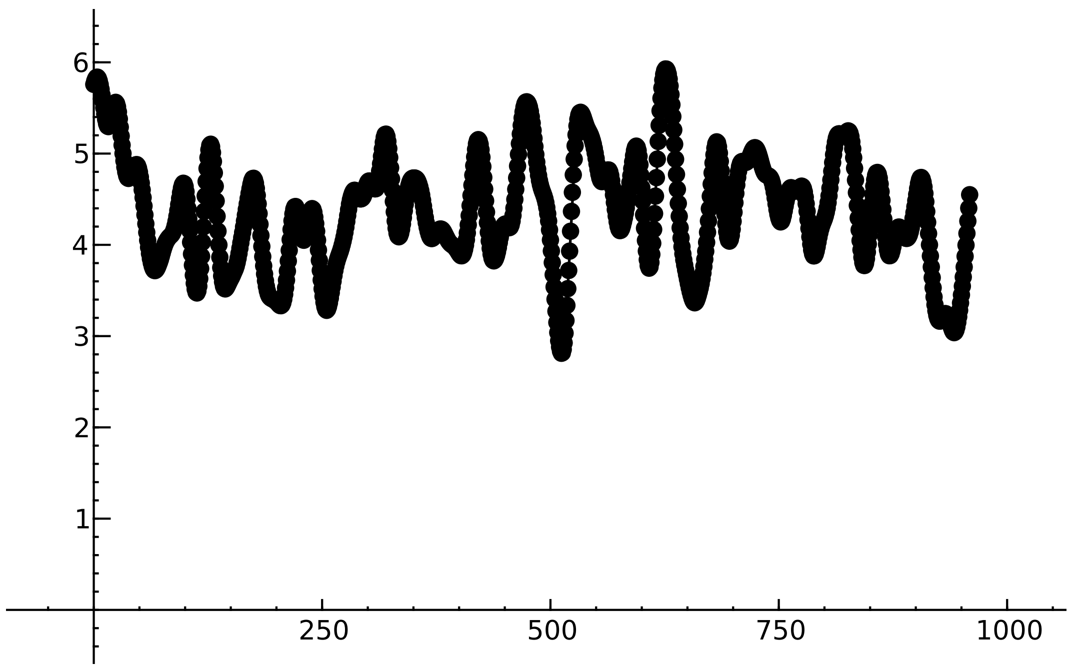

Figure 5: Sequence from four moving averages.

References:

- Barnett, 2006. "Chancing an interpretation: Slutsky's random cycles revisited ," European Journal of the History of Economic Thought, Taylor and Francis Journals, vol. 13(3), pages 411-432, September.
- Royama, 1992. Analytical population dynamics. Springer.
- Slutsky, 1937. The summation of random causes as the source of cyclic processes. Econometrica 1937;5:105-46