Euro. J. History of Economic Thought 13:3 411–432 September 2006 Routledge Taylor & Francis Group

Chancing an interpretation: Slutsky's random cycles revisited*

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1. Introduction

Sometimes the meaning of a groundbreaking contribution in economics is crystal clear and completely unambiguous immediately on first publication; sometimes it is not. Sometimes a specific interpretation of the revolutionary ideas of an economist gains ground right from the start, is then widely disseminated as 'what X really meant', only sometime later to be revealed as an overly simplistic or even misleading interpretation of X's true meaning, Keynesian IS-LM analysis being a classic example. In still other instances, whilst an article is almost universally recognized as being significant, the precise meaning and relevance of it is not always immediately apparent, or remains essentially contested terrain for some considerable period of time.

E.E. Slutsky's article 'Slozhenie sluchainykh prichin, kak istochnik tsiklicheskikh protsessov' – translated as 'The summation of random causes as the source of cyclic processes' – is universally recognized as being a very important contribution to the development of the analysis of economic fluctuations (Slutsky 1927).[1] It is frequently described as 'groundbreaking'. It was first published in *Questions of Conjuncture* (*Voprosy kon"yunktury*), the theoretical journal of the Moscow Conjuncture Institute, a publication that had been first issued in 1925. Despite its initial Russian language form, it was quickly spotted as being worthy of serious attention by American economists such as Wesley Mitchell and Simon Kuznets and received prominent English-language distribution ten years after its first

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The European Journal of the History of Economic Thought
ISSN 0967-2567 print/ISSN 1469-5936 online © 2006 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/09672560600875596

appearance, translated in *Econometrica* (Slutsky 1937). As noted much later, in its most extreme construal, Slutsky's demonstration admitted of the interpretation that the business cycle was nothing but a statistical artefact (Kim 1988: 1).

A short summary of the underlying concept that is often understood by many economists to have been developed in Slutsky's 1927 article can be given as follows. If the variables that were taken to represent business cycles were moving averages of past determining quantities that were not serially correlated – either real-world moving averages or artificially generated moving averages – then the variables of interest would become serially correlated, and this process would produce a periodicity approaching that of sine waves.[2] By way of some contrast to this type of approximate summary, the following article examines Slutsky's ideas in detail as they are found in the original article and in some closely associated publications.

R.G.D. Allen suggested that Slutsky's results were of great value in researching whether a moving average trend distorted the true oscillations in a series, and were also significant regarding the structure of economic time series themselves (Allen 1950: 210). However, as will become apparent from what follows, Slutsky's 1927 article was subject to some variant readings, readings that were not always totally contradictory, but certainly not completely unambiguous. Moreover, various streams of propagating influence can be traced outwards from the original article, which might have generated ideational cycles of variant effect, some perhaps spurious, some not so. Even today, dispute about 'what Slutsky really meant' is still very much a live issue in the economics profession. For example, in a debate that occurred in 1997 between Milton Friedman on the one hand and F.E. Kydland and E.C. Prescott on the other, Friedman questioned whether models containing technological shocks that mimicked cyclical behaviour could be said to actually explain this behaviour (Friedman 1997: 210). It was Slutsky's 1927 article that Friedman referred to in detail to make the case on this point.

Before proceeding further, however, it is necessary to acknowledge that the 1937 *Econometrica* English-language version of the article was an expanded version of the 1927 original, a new section being added towards the end of the article.[3] This addition involved something that Slutsky had first published in 1929 in Russian, a paper on the standard error of the correlation coefficient applied to the case of a coherent chance series (Slutsky 1929). Whilst this addition certainly did not radically change the nature of the original version, it did suggest that Slutsky had continued to work on related topics after 1927.[4] This possibility is further investigated in what follows.

2. The two basic propositions of the 1927 article

Slutsky identified the 'basic problem' that the 1927 article had set out to investigate as follows: was it possible that a connective structure between random fluctuations could transform them into a system of approximately regular waves (Slutsky 1937: 106)? In order to answer this question Slutsky had attempted to prove two separate but related propositions as follows:

- Proposition number one – the summation of random causes might generate wave-like phenomena, i.e. that mutually independent chance events might conjoin together to produce an oscillatory appearance in some aspect of reality that was represented in a time series-like fashion (ibid.: 114).
- Proposition number two – these wave-like fluctuations might imitate cycles exhibiting approximate regularity (i.e. rough periodicity), at least for a definite period of time (ibid.: 118).

Note that it was logically possible to prove proposition one but then fail to prove proposition two, but not vice versa. However, if Slutsky had managed only to prove proposition one, then the relevance to economics was greatly reduced, as the prize of explaining periodic trade cycles would be lost. Thus, proposition two was a more far-reaching and radical idea than proposition one.

The significance of these two propositions for economics was that if Slutsky was right in both cases, then actually observed business cycles could possibly be explained as the summation of purely random causes and did not necessarily require explanation by means of any periodic underlying mechanism such as Karl Marx's replacement time of basic investments, W.S. Jevons's sunspot cycles linked to weather patterns, the duration of capital goods or the periodic gyrations of credit policy. Another veiled possibility that was not fully articulated in the original article was that randomly generated cycles might be indistinguishable from cycles with periodic causes, in that they both would appear to an observer as very similar, if not actually identical, although both were (in this scenario) actually present in a given economy.

However, even if the logic of Slutsky's reasoning (as presented in the article) was accepted as valid for both propositions, he had demonstrated only that it was logically possible for regular business cycles to have random causes, not that they actually did. In the article Slutsky superimposed two graphs, one showing a statistically generated 'random' cycle and the other an actually observed business cycle, these two graphs coinciding quite dramatically. But this visual comparison did not prove conclusively that actual business cycles were randomly generated, as the similarity could have been mere coincidence. Slutsky himself was clear on this deficiency, writing that the comparison was a 'graphic demonstration of the possible effects of the summation of unconnected (nessvyaznyi) causes' (Slutsky 1937: 110), with the covert stress on 'possible'.[5] Thus, the implication that actual business cycles might be randomly generated was left open for readers to ponder and develop, whilst Slutsky himself moved away from economics-related work due to the dark clouds of Stalinism that were gathering in the USSR at the end of the 1920s. However, as will be seen further on, Slutsky continued to work on chance processes from a mathematical and statistical perspective after 1927, leaving the economic component to be taken up by economists elsewhere, if they so desired.

3. The 1927 article in more detail

Again, Slutsky's basic thesis was that the summation of random causes could generate a cyclical pattern in a time series, which would imitate for a number of cycles a harmonic series, but this imitation would not last forever. After a number of periods this cyclical pattern of 'random' cycles would become disarranged and the transition to another regime (series pattern) would occur around certain critical points. Fundamental to Slutsky's analysis of this phenomenon was harmonic analysis, or the expression of the irregularities in the form and spacing of real cycles in terms of the summation of a number of separate regular sinusoidal fluctuations, an area of mathematics that had been developed by Jean Baptiste Fourier.

Regarding proposition one, Slutsky reasoned that the probability of a value in a series remaining above or below the trend for a long period was negligible; hence, values would pass from positive to negative deviations from the trend quite frequently. This would inevitably give rise to an undulatory appearance (Slutsky 1937: 114–7). However, this cyclical pattern might not be periodic. In order to prove proposition two, Slutsky had to provide a second stage of argument, which was much more complicated than the first. This second stage involved the idea that the decomposition of a wave pattern into various Fourier series revealed with greater distinction the regular wave pattern desired.[6] For example, if a curve was represented as the product of two sinusoids, then these sinusoids, according to Slutsky, separated on a graph the regions that corresponded to definite regimes or series patterns. The point where such a sinusoid cut the axis of the abscissa was the critical point beyond which one regime was replaced by another one with different parameters.

Slutsky related that any given curve could be represented by the sum of a series of sinusoids, provided a large enough number of terms was taken. However, not every empirical series yielded a clear periodicity with only a small number of harmonics being employed. When a given series did do so, the explanation was found in the mechanism of connection of the random values involved. The partial sections of approximately regular waves apparent in the crude series that Slutsky presented were made much more distinct when the sum of the first five harmonics were deducted (Slutsky 1937: 118–23). Slutsky took this as evidence that random waves could contain sections of apparently periodic movement following Fourier analysis.[7] As this concept was summarized by Slutsky – the summation of random causes generated a cyclical series that tended to imitate for a number of cycles a harmonic series of a relatively small number of sine curves (ibid.: 123).

A key feature of the 1927 article was a number of statistical experiments in which the processes of random summation were modelled. In these experiments Slutsky applied various different moving average and differencing procedures to sequences of random numbers as follows. First of all he took a ten-item moving summation of the first basic series of random numbers, and it was the graph that resulted from this procedure that was juxtaposed to the index of English business cycles for 1855–77. Slutsky also performed a two-item moving summation twelve times in succession on the third basic series of random numbers, and then took first and second differences of the results. The graphs of these multi-summated and then differenced sequences might also have looked quite similar to actual plots of business cycles, if they had been so compared. Even more significantly, Slutsky applied a tenfold summation of three items at a time, with the weights chosen at random for each successive summation. When depicted graphically by means of ten separate charts, this procedure could clearly be seen to approach the Gaussian curve as a limit, a striking visual illustration of the tendency to sine wave form through repeated moving summation that Slutsky had revealed.

4. Interpreting Slutsky's 1927 article

One of the main consequences of this work of Slutsky's in the economics arena was taken to be that an oscillatory series could be generated from a random series by taking a moving sum or difference; that is, if a moving average of a random series was taken, for example, to determine trend, then a (spurious) oscillatory movement in the series might be generated where none had existed originally. Put another way, if a moving average of an 'incoherent' (Slutsky's term for serially uncorrelated) series was taken, then a 'coherent' (Slutsky's term for serially correlated) series could be obtained. A similar characteristic was discovered independently by G.U. Yule, and it is usually referred to as the Slutsky–Yule effect (Yule 1926).[8]

This effect was related to but was not identical with the idea that the summation of random causes could be the source of actual cyclical tendencies in the economy, these fluctuations displaying approximate regularity. The fact that Slutsky was suggesting that real processes in the economy might be modelled as random summation is apparent from the following passage:

> ...it seems probable that an especially prominent role is played in nature by the process of *moving summation* with weights of one kind or another... For example, let causes x_{i-2}, x_{i-1}, x_i, produce the consequences y_{i-2}, y_{i-1}, y_i, where the magnitude of each consequence is determined by the influence, not of one, but of a number of the preceding causes, as for instance, the size of a crop is determined, not by one day's rainfall, but by many.
>
> (Slutsky 1937: 108).

The same indirect relation to real economic processes applied to the sinusoidal limit theorem, which stated that the summation of random causes could generate a specific sine wave (Gandolfo 1987, vol. 4: 356).

These two interpretations of Slutsky's results are sometimes confused, in that there was both a 'real' and a 'statistical' interpretation of the 1927 article, a distinction that might also be termed a 'genuine' as opposed to a 'spurious' interpretation. Did the processes involving moving summation that Slutsky identified as occurring in nature extend to developments in the economy (the 'real' or 'genuine' interpretation), or was this effect limited to the manipulation of constructed time series by investigators (the 'statistical' or 'spurious' interpretation)? It could be argued that in the 1927 article itself Slutsky favoured the 'real' or 'genuine' interpretation, as the previous quotation linking rainfall over many days to crop yields implies. On the other hand, the footnote reference to Yule's work on nonsense correlations might be taken to suggest that the 'statistical' or 'spurious' interpretation was also being legitimated. Later commentators have tended to emphasize one interpretation as opposed to the other, and sometimes even to shift from one interpretation to the other over time.

Slutsky's work on the random causes of cycles was not only relevant to economic affairs, but to all activities in which the periodicity of time series was involved. Thus, it had direct relevance to all statistical manipulations of data, whether economic, meteorological, mathematical or in any other field. Slutsky's analysis also implied that a time series might contain structural breaks, in which a regularity that had previously acted on the series was replaced by a new regularity with different parameters (Slutsky 1937: 105), this being called a regime change.[9]

Slutsky's investigation into economic fluctuations was very different in approach to both Nikolai Kondratiev's in the USSR and Wesley Mitchell's in the USA at this time. Slutsky argued that Mitchell's denial of the periodicity of business cycles was a result of Mitchell's purely descriptive methodology. Using Slutsky's more sophisticated Fourier-series analysis, periodicity was discovered, albeit confined to definite regions of a series. Slutsky named the tendency of random series to possess periodicity 'the tendency to sinusoid form' (ibid.: 126). Therefore according to Slutsky a key difference between randomly generated cycles and genuinely periodic cycles was that in the former periodicity would not extend to the entire series under consideration, whereas in the latter it would.

The uniqueness of Slutsky's approach within the USSR is emphasized by examining the range of authors on which his work drew. Slutsky footnoted Arthur Schuster's articles, 'On the periodicities of sunspots', and 'The Periodogram of the magnetic declination as obtained from the records of the Greenwich Observatory'; L. von Bortkiewicz's work on iteration; E. Husserl's work on the philosophy of phenomenology; V. Romanovsky's work on the sinusoidal limit theorem; W. Thorp's business annals; R. von Mises on probability; K. Pearson's statistical tables; and A. Lyapunov's analysis of the limit properties of integrals. This incomplete list demonstrated that Slutsky's theoretical approach to cycles was not really influenced to any large extent by the more conventional economics authors, with which other Conjuncture Institute members were more familiar, people such as M.I. Tugan-Baranovsky, S.A. Pervushin, G. Cassel, A. Pigou, A. Aftalion and so on.[10] The impetus for the new approach of the 1927 article had originated from Slutsky himself.

5. The Kuznets elucidation

One of the first papers written in English to discuss Slutsky's ideas on the random causes of cycles in any detail was 'Random events and cyclical oscillations' by Simon Kuznets, which was published in the *Journal of the American Statistical Association* in September 1929. Kuznets was a member of Mitchell's National Bureau of Economic Research at this time, and (given his country of origin) could read Russian very well. In this paper Kuznets highlighted the 'shock' impact of what Slutsky had written. Kuznets wrote: 'It is not only ''possible'' that a summation of a random series will yield cycles, but also quite certain that this will be the case. Indeed this is the first thesis of a remarkable memoir by Professor Eugen Slutsky...' (Kuznets 1929: 258).

In his 1929 article Kuznets conducted his own tests of Slutsky's idea by subjecting random data series to statistical manipulations. According to Kuznets, while the original series did not show the appearance of cycles, the manipulated series indicated clearly the desired oscillations.

Kuznets explained this phenomenon as follows. While the successive items of a chance distribution were entirely uncorrelated, the successive items of a cumulation or a moving average, having a number of items in common, were closely correlated. The larger the number of items included in a moving average, the closer the correlation might be expected to be. Kuznets further outlined that the line resulting from a moving average ran in waves because there were clusters of positive and negative deviations from the average in the distribution of random causes. Hence, with a series of many items it was unlikely not to contain such 'cycles' around the average. This latter idea was directly from Slutsky's 1927 presentation.

However, Kuznets added an original statistical idea of his own, which was that an extremely large deviation from the mean, since it would be included in a number of members of the moving average, would tend to raise or depress the level of all the members that included it, and, hence, would tend to form cyclical swings. Kuznets concluded that the shape of the distribution of the random causes and the period of the moving average would influence the amplitude and timing of the cycles thus generated. He suggested that a peaked distribution skewed to one side was the most likely source of clear-cut cycles (ibid.: 263–73).

At the end of the paper Kuznets was cautious that the inversion of inference from random events causing cycles to actual cycles being caused by random events could not be made with certainty. He suggested that this idea should be tested as a hypothesis against the known facts of cyclical oscillations. If proved correct, however, the whole discussion of the causes of business cycles would become supererogation, since cyclical swings were bound to occur sooner or later as the result of the accumulation of random events. Kuznets took this as confirmation of the institutional approach to explaining business cycles, which was concerned with the economic forces that make for cumulation, and with forces that explained why a random event was not cancelled by an opposite reaction but was allowed to exert its influence for some time to come. The institutional approach was thus for Kuznets an economic counterpart of the statistical method of the moving average (ibid.: 275). This contrast explicitly paralleled the real cyclical process occurring in an economy modelled as a moving average procedure itself (the 'real' or 'genuine' interpretation of Slutsky's article), with the spurious cycles generated by some types of statistical manipulation of data series (the 'statistical' or 'spurious' interpretation).

Kuznets also added an original economic understanding to Slutsky's argument as follows. Kuznets hypothesized that, if smooth cyclical oscillations were observed in a data series, such as those seen in wholesale price indices or indices of production, then this smoothness could be conceived as the result of repeated summations of random events over some significant period of time. If, on the other hand, a choppy, highly fluctuating series like the number of shares sold on the New York Stock Exchange was witnessed, then this might be ascribed to the short period during which the chance events were allowed to operate (ibid.: 275). Hence, Kuznets went beyond Slutsky by suggesting a way of distinguishing the length of time random events had in which to be summated in various different types of economic processes that were represented in time series data.

6. The Frisch elucidation

Ragnar Frisch appeared initially most interested in the econometrically relevant element of Slutsky's conclusion (the 'statistical' or 'spurious' interpretation), naming the Slutsky effect as the fact that linear operations applied to a random variable might produce fluctuations of a cyclical character (Frisch 1995a, vol. 1: 189). The relevance to understanding actual economic cycles was not emphasized by Frisch in his first discussion of Slutsky's work in 1931. Here Frisch suggested that an understanding of the laws of spurious cycle creation would assist in eliminating them, something that could be accomplished by setting aside one root of a key equation to take up the spurious effect. As Frisch was (in 1931) concerned with eradicating spurious cycles, he could not have been fully attuned to the idea that actual business cycles might be explained by random summation (the 'real' or 'genuine' interpretation).

However, two years later, in 1933, Frisch explained that Slutsky had definitively established that some type of swings would be produced by the accumulation of erratic influences, but Frisch noted that the general law setting out what sort of cycles that a given kind of random accumulation would create had not yet been outlined (Frisch 1995b, vol. 1: 339). According to Mary Morgan, Frisch felt that the central problem of how Slutsky's random events came to be absorbed into the economic system remained to be answered (Morgan 1990: 96). This was certainly a correct interpolation, as Slutsky himself was not really qualified to answer this type of question, his knowledge of 'conventional' business cycle theory being somewhat restricted. Slutsky knew his own limitations. In fact it was Frisch's goal in his 1933 article on 'Propagation problems and impulse problems' to explain exactly how Slutsky's random events came to be summed by the economic system (ibid.: 92), and Knut Wicksell's mechanistic analogy with a rocking horse being set into motion by random shocks was cited in this respect.[11]

Later historians have suggested that it was Slutsky's 1927 article that helped Frisch to construct a mathematical model of the trade cycle in which the oscillations were caused by exogenous shocks (Beaud and Dostaler 1995: 65). In reality, Slutsky had not described the random events that might sum to generate cyclical patterns as unquestionably 'exogenous' or 'external' to the system under review. In fact, a plausible interpretation of the 1927 article might be that the chance events occurred within the process under consideration; using the distinction exogenous/endogenous with regard to economic theory was a somewhat later invention. The same might be seen to apply to A.H. Hansen's distinction between impulses that initiated periodic movement and conditions that determined the way in which an economy responded to such impulses (Hansen 1951: 363). Such a distinction can be read into Slutsky's approach, but it is also possible to conceive of Slutsky's random cycles as an integrated whole.

7. Further deliberations on Slutsky's 1927 article

Other notable contemporary figures (apart from Kuznets and Frisch) to take up Slutsky's initial challenge were Gerhard Tintner, Edward Dodd, Harold Davis and Jan Tinbergen. Tintner pointed out that Slutsky's theory lacked any explanation of the mechanism through which small random variations were 'summed' in the economic process and hence the apparent visual similarity of actual cyclic variations and summated random series should be regarded as 'merely spurious and superficial' (Tintner 1938: 148).[12] Dodd suggested that the length of random cycles should be conceived as the reciprocal of relative frequency: if the probability of a value being a maximum was 0.05, then five maxima per hundred values should be expected, making the cycle length equal to twenty units (Dodd 1939: 255–6). Davis discussed Slutsky's 1927 article in the context of analysing the theory of serial correlations and was one of the few early commentators to mention the sinusoidal limit theorem (Davis 1941: 57).

Discussing the effects of using moving averages without mentioning Slutsky by name, Edwin Frickey outlined that the average length of fictitious cycles thus generated increased regularly as the period of the moving average was lengthened (Frickey 1942: 47). P.A.P. Moran provided additional proof of Slutsky's sinusoidal limit theorem, highlighting that both moving averages and differencing of the random series were employed in order to obtain convergence in probability to a sine wave (Moran 1950: 272).[13] In his textbook *Econometrics*, Tinbergen suggested that the most important economic interpretation of Slutsky's 1927 article was that the random shocks involved were 'changing crop yields and a few very important political events (e.g. wars)' (Tinbergen 1951: 151). However, Tinbergen did not reject completely the idea that those constructing econometric models of business cycles should also consider 'more systematic causes', i.e. non-random factors.

It is perhaps surprising to realize that, immediately following the appearance of Slutsky's work, no one apparently attempted to prove directly whether economic cycles really were the result of the summation of purely random causes or were the result of real periodic factors. Frisch's primary concern was with how random factors were summated, not with the question of whether 'real' business cycles were thus generated. A third possibility, not always recognized, might also be outlined, that economic cycles were the consequence of some truly periodic influences, compounded by the action of the summation of some random elements. A fourth possibility (already mentioned) was that two different types of business cycle were found in market economies, one randomly generated, the other not so.

One of the basic problems revolved around the nature of the phenomenon under investigation. Economic cycles were fluctuations in the business activities of human subjects, but such activity could only be measured through time series data relating to specific variables such as prices, interest rates, exchange rates and so on. Hence business cycles were being detected by means of statistical analysis applied to time series data. But Slutsky had suggested (or was interpreted as suggesting) two different things, that some types of statistical manipulation applied to time series data might produce the appearance of pseudo-cycles in the resultant data series (the 'statistical' or 'spurious' interpretation), and also that the economic cycles that were the underlying subject of the investigation might themselves be the result of the summation of chance factors in the real world (the 'real' or 'genuine' interpretation). If random cycles were then found in the analysed data sets, were they the result of statistical manipulation, or summed random factors that were real? It might be possible to distinguish in theory between 'real' random cycles and statistically generated spurious cycles, but how could this distinction be detected in practice? Also, even if it was possible to rule out the appearance of statistically generated pseudo-cycles by some completely reliable method, how could it be decided if the remaining cycles were the outcome of summed random non-periodic factors, or real periodic influences?

One point of significant ambiguity remained concerning the length of time that a randomly generated fluctuation might continue to be propagated before the transition to a new regime pattern (a regime change) occurred. Might randomly caused cycles in an economy last for weeks, months or even years? That is, of greatest relevance to economic theory, could the Mitchell-style business cycle of eleven-year duration be the result of the summation of purely random causes? Slutsky passed no final judgement of this question in the 1927 article, but it was obviously of the greatest importance for the development of business cycle theory after him. The question becomes even more controversial when it is realized that, for the eleven-year periodicity to be thought about in this way, the repetitions required for the regime pattern to be perceived as remaining constant meant that a time-span of many decades was really being suggested as requiring consideration. In relation to this, the question of why specific sinusoid regimes in the economy became disrupted should also be raised. Might such a periodic disarrangement have real economic causes, or be purely a consequence of the statistically conceived process of the summation of random elements?

Mitchell admitted in 1927 that Slutsky's first proposition had a bearing on his own inference from the distribution of cycle durations (Mitchell 1927: 478). Mitchell suggested that any dominant factors producing uniform duration were greatly compounded by many other lesser factors (ibid.: 420). In *Measuring Business Cycles* of 1946, Burns and Mitchell did not mention Slutsky by name, but they did consider the effects of random elements. They wrote:

> In historical series the effects of cyclical and random forces cannot be separated even over the course of a full cycle. Random factors constantly play on business at large and on each of its many branches, and their effects register in different ways under different circumstances.
>
> (Burns and Mitchell 1946: 320).

This suggestion that it was not possible to distinguish cyclical and random forces was of direct relevance to Slutsky's approach, as the 1927 article had implied that such a distinction was crucial to understanding the underlying mechanisms generating economic cycles.

Finally, an ambiguity runs throughout Slutsky's 1927 article at a philosophical level as follows. On the one hand, against economists such as Mitchell, Slutsky was concerned to demonstrate the approximate periodicity of business cycles, rather than just their non-periodic rise and fall. But in order to do this, he resorted to arguing that actual business cycles might be generated by the summed effect of non-periodic factors. Mitchell might have responded that, yes, random causes might generate real cycles in some instances, but in other cases they might not. It just all depends.

8. What is a 'random cause'?

Another point of possible ambiguity concerned the nature of the random events under consideration themselves. Did Slutsky mean 'random' only in the sense of being non-periodic but continuous, or non-periodic and unrelated? By this distinction is meant the same type of cause occurring many times but non-periodically, or a whole number of separate causes occurring non-periodically? Also might the random events that Slutsky was modelling be of a type that economists had previously discussed – monetary disturbances, the gestation of investment programmes and so on – or might the chance factors in question be of a totally new type not previously considered by economists at all – truly random events such as absenteeism from work due to illnesses or extreme weather events causing localized disruption to business? Such issues were raised by subtle implication in the 1927 article but left open to variant interpretation.

The concept of randomness itself also deserves further consideration. Dictionary definitions of random cite 'having no specific pattern' and 'a phenomenon that does not produce the same outcome every time it occurs'. One economist suggested that the concept of 'random' related to an influence that did not recur (Hald 1954: 15). A random variable in a statistical sense is one having numerical values that are determined by the results of a chance experiment. But what are 'random causes'? On first view they are causal influences that are either random events themselves or are the result of random events. Considering another translation of the Russian phrase 'sluchainyi prichin' ('accidental causes'), attributes that are not essential to the nature of something come to mind. It was Slutsky who was instrumental in developing the term 'stochastic processes' as a synonym for 'random events', but did he have some special meaning in mind in this case?

Sometime later the term 'random causes' was transformed into 'random shocks' by some commentators. For example, in an article entitled 'Business cycles – endogenous or stochastic?', Irma Adelman suggested that 'the primary task of the business cycle analyst is to investigate the reaction patterns of an economic system to various shocks' (Adelman 1960: 795). Adelman had previously declared that: 'The idea that economic fluctuations may be due to random shocks was first suggested in 1927 by E. Slutzky…' (Adelman and Adelman 1966, 288: fn.19). Some commentators have even gone as far as identifying something called the 'Frisch-Slutsky' approach, in which shocks to an economy's given equilibrium path were posited as the causes of cycles. In fact Slutsky had never used the word 'shocks' in his original discussion, and it was Frisch (not Slutsky) who had referred to 'erratic shocks' as the energy source maintaining oscillations (Frisch 1995b, vol. 1: 337). This illicit terminological substitution was adopted by later economists such as Tinbergen and those of the real business cycle school, without fully realizing that the meaning of Slutsky's original article was being distorted.[14] Moreover, Slutsky did not draw upon neoclassical notions of equilibrium either in his work on the random causes of cycles, an illicit addition that sometimes accompanied the illicit substitution.

9. The context of the Moscow Conjuncture Institute

Now that the content of the 1927 article has been considered in some detail, the context of its creation deserves further attention. Slutsky had transferred from Kiev to Moscow in 1926 in order to take up Kondratiev's offer to become a consultant in the Conjuncture Institute, which was part of the People's Commissariat of Finance at this time. Hence, the 1927 article was the direct result of Slutsky coming into contact with the work in economics being conducted by Kondratiev and his colleagues in Moscow, and it is worth exploring the possibility that some mutual influences might be detected in this respect.

The culture of the Conjuncture Institute was mainly (although by no means exclusively) empirically orientated, with Kondratiev being primarily an agricultural economist by training. However, members such as A.A. Konyus and N.S. Chetverikov provided some significant theoretical and statistical input, and in a footnote to the 1927 paper, Slutsky acknowledged the help received from a number of assistants. The Conjuncture Institute itself had separate sections devoted to the methodology of conjuncture, headed by Chetverikov, and to indices and prices, headed by Konyus, and both of these section leaders later made notable contributions to statistics.

More significantly, Slutsky was not the only economist in the Conjuncture Institute to recognize the use of Fourier analysis in understanding economic cycles. The work of Western economists such as H.L. Moore in the area of periodogram analysis was well known in Kondratiev's centre, as shown by the following review. One of the leading members of the Conjuncture Institute, A.L. Vainshtein, discussed two of Moore's books in the Institute journal in 1925: *Economic Cycles: Their Law and Cause* of

1914 and *Generating Economic Cycles* of 1923. Moore had connected meteorological patterns to economic cycles by associating rainfall with harvest. According to Moore periodic meteorological fluctuations had a decisive effect on the production of goods dependent on climatic and natural conditions, namely raw materials (Vainshtein 1925: 165). Moore, like Slutsky, had used Fourier analysis to search for cyclical patterns. However, Moore used a slightly different technique than Slutsky.

Vainshtein related that in order to distinguish real cycles from spontaneous random cycles, Moore had used the periodogram method, which was developed by Schuster in his work on meteorology. This involved calculating the square of the coefficient of the first member of the Fourier harmonic series; that is, taking the square of the amplitude of the first harmonic. A periodogram graph was then constructed in which the abscissa was the number of years and the ordinate was the amplitude squared. Comparing the actual level of the square of the amplitude to the average level for the entire series, a way of detecting the period of actual cycles was found: by finding peaks at certain years far higher than the average. Using these techniques Moore had found the periodicity of economic cycles calculated from A. Sauerbeck's price index to be 96, 48, 19.2, and 8 years (ibid.: 168–70).

In the conclusion of his review, Vainshtein criticized the methods used by Moore for a 'purely formal, statistical analysis of the question' and for ignoring the economic meaning of the problem. Vainshtein also criticized Moore for his inability to provide a mechanism of influence for the connections he assumed in the statistical methods utilized (ibid.: 179). It is apparent that Slutsky was not the only member of the Conjuncture Institute who was interested in the statistical analysis of time series data relating to economic cycles, although Vainshtein's attitude suggested he might have been similarly critical of Slutsky's effort. Even so, it is a distinct possibility that Slutsky had first realized that such statistical techniques might have been relevant to economic matters through exposure to such discussion in the Conjuncture Institute.

10. Slutsky's work before and after 1927

In terms of further understanding the set of interests flowing into the 1927 article, it is worth discussing Slutsky's relevant work before 1927. In the 1920s Slutsky worked on a number of related topics in mathematics and statistics, such as the law of large numbers and on various limit theorems, and also on the foundations of probability theory. In particular, Slutsky created the concept of the stochastic asymptote or limit, which was later of direct use in developing econometric theory. Slutsky suggested that between the stochastic and the usual understanding of the limit was significant logical gap, across which without an understanding of probability there would be no bridge (Slutsky 1960: 285). What Slutsky accomplished that was original was to obtain a new sequence of random variables by subtracting the expectation (theoretical probability) of a random variable from each actual value of the variable, the sequence thus obtained being required to converge to zero, if the stochastic limit was seen to apply. In equation form:

$$X_n - EX_n \to 0$$

Put another way, for stochastic convergence to be observed, the difference between the arithmetic average of the expectation and any particular value of the probability would tend to zero as the number of possible outcomes increased.[15] Slutsky's own contribution to limit theory was provided in the context of a strong and prolonged interest in the mathematics of probability amongst Slutsky's Russian contemporaries, people such as A.A. Markov and A.A. Chuprov, who were leaders on the international stage in these subject areas.

Slutsky devoted a long and substantial article to the topic of the stochastic limit in 1925 (Slutsky 1925), and he also discussed limit theorems in the 1927 article on random cycles. Moreover, Slutsky continued to work on stochastic convergence after 1927, writing an article on the relevance of this topic to random quantities in 1928, and another article on several propositions relating to the stochastic limit in 1929. Thus, Slutsky had been thinking about stochastic processes for at least two years prior to the 1927 article, if only in a formal mathematical sense, and on probabilistic processes in general since the early 1920s. The impetus to apply this type of analysis to economics obviously came from within the Conjuncture Institute, although none of Slutsky's colleagues appeared to take up the challenge by attempting to continue the same line of investigation after 1927.

In the event, the 1927 article was Slutsky's final direct contribution to economics. As already noted, he did however continue to work on some of the mathematical and statistical themes articulated in the 1927 article after this date, as they were of no immediate political significance and hence were unlikely to provoke any direct concern from Communist Party personnel.

An example of something at least indirectly related to the 1927 article published by Slutsky in 1935 was a paper entitled 'On the question of extrapolation in connection with the problem of prognosis', which was published in the Soviet *Journal of Geophysics*. In this article, Slutsky investigated the use of the extrapolation method in relation to determinate random processes, suggesting that the effectiveness of this method depended on the probability structure of the process under review. The possibility of extrapolation was proved by the existence of limiting cases such as the law of the sinusoidal limit, and Slutsky referred to his 1927 paper in this respect directly. After describing various methods based on a number of different regression equations, Slutsky recommended that the best way of testing a particular method was by comparing it with a real application. He thus presented as a practical example a prognosis of the water level of the Volga River at Saratov over ninety-five days in 1925, outlining that an accurate forecast for two days ahead was possible through the use of a specific equation that was given (Slutsky 1935: 274–5). While in no sense as significant for economics as the 1927 article, Slutsky's work in the 1930s was in a limited sense a continuation of the themes that were articulated there.

11. Slutsky's influence on later economists

In terms of the lasting influence of the 1927 article on the use of statistics in economic theory, Slutsky's contribution has been documented thus:

> The main tradition in time-series modelling was founded by Wold (1938), who established a link between the autoregressive (AR (p)) and moving average (MA (q)) formulations of Yule and Slutsky, respectively, and the probabilistic structure of stochastic processes formalized by Kolmogorov and Khintchin.
>
> (Spanos 1990: 339)

In fact, as has been documented above, Slutsky also made a (relatively minor) contribution to the latter topic, being a colleague of Kolmogorov and writing on the law of large numbers and limit theory in general throughout the 1920s. Hence Slutsky's role in the development of econometric modelling was significant, and was not limited to the 1927 article alone.

In terms of Slutsky's influence on the more recent development of trade cycle theory, in the 1980s a novel approach to explaining cycles called real business cycle analysis was developed in the USA. One of the authors of the classic paper 'Time to build and aggregate fluctuations', E.C. Prescott, explicitly acknowledged that Slutsky's 1927 article was (in part) an inspiration for this approach. Prescott outlined that he did not like to use the term 'business cycle' because some systems of stochastic equations with a non-oscillatory component displayed cycle-like features, a fact that was directly attributed to Slutsky's 1927 article (Prescott 1998: 84). Moreover, Kydland and Prescott also acknowledged the unequivocal influence of Slutsky on Robert Lucas's definition of business cycles as 'co-movements of the deviations from trend in different aggregate time series' (Kydland and Prescott 1998a: 387). Lucas himself has succinctly encapsulated Slutsky's contribution to his own intellectual development through remembering the revelation that he had on first encountering the article as being: 'Hey, this thing looks like pictures I saw in Mitchell's book' (Lucas 2004: 22).

Kydland and Prescott raised the question of 'how to determine which sources of shocks give rise to cycles', (Kydland and Prescott 1998b: 226), something that (if 'shocks' were replaced by 'causes') Slutsky's work had begun to explore. In answer to this question, real business cycle theory posited that real shocks such as productivity changes or the time required to construct investment goods initiated the propagation of economic fluctuations, and if these real factors were non-periodic in appearance, they might be thought of as Slutsky's random causes that generated actual cycles. However, as was noted previously, Slutsky never used the term 'shocks' to describe the random causes being summated through time, and the notion of random shocks being dampened likely owed more to Frisch that to Slutsky. Even so, it is apparent that some elements of Slutsky's economic ideas came back into vogue approximately fifty years after their first propagation.

12. Conclusion

An attempt to highlight the development of various readings of Slutsky's 1927 article has been made, such as the 'statistical' or 'spurious' interpretation vs. the 'real' or 'genuine' interpretation, together with explaining some of the conjunctural and contextual elements that contributed to its creation. How some contemporary and later economists and statisticians have elucidated Slutsky's work was also discussed. From this presentation it can be seen that Slutsky's 1927 article was open to a number of different interpretations that varied (in part) in relation to the particular concerns of the economist and/or statistician undertaking the interpretation.

It is also apparent that the personal and professional processes that produced the 1927 article were rather complex, with a number of separate elements combining successfully in the right place, the right time and the right mind. Hence it is quite possible that if Stalin's 'left turn' in 1929 had occurred only a few years earlier, then Slutsky might well have been discouraged from applying his mathematical and statistical understanding to economics, with the result that the 1927 article would have remained unwritten. Exactly what additional economics-relevant work that Slutsky might have embarked upon after 1929 if Stalin had failed in his attempt to become all-powerful in the USSR is impossible to accurately predict.

Notes

* I am grateful to the comments of two anonymous referees for some suggested improvements and clarifications on an earlier version of this article. One referee's comments were particularly detailed and I thank them especially for their assistance.

1 An alternative translation of the Russian title might be 'The Compounding of Accidental Causes as the Origin of Cyclic Processes'.

2 I am grateful to one of the referees for the basic outline of this summary.

3 Some of the original data series given in the 1927 article were excluded from the 1937 version.

4 For some background information on Slutsky's life, see Barnett (2004).

5 It is necessary to discuss this procedure in some more detail. Presenting a graph that coincides quite accurately with actual business cycles, but is generated by summed random causes, neither proves nor disproves any sort of explanatory link between the two. If the graph coincides better than any other model currently available, it might be accepted as the approach that currently 'best fits' the data, but this does not mean for certain that it is completely accurate. At any point in the future an even better model could replace it, or a less accurate model could replace it with a more plausible connection to the actual events being described.

6 In Fourier analysis a waveform is analysed to discover the sine wave frequencies that it contains. Through harmonic analysis it can be shown that periodic non-sinusoidal waveforms are composed of combinations of pure sine waves. One major component, a large amplitude sine wave of the same frequency as the wave under consideration, is called the fundamental. The other components are sine waves with frequencies that are exact multiples of the frequency of the fundamental. These harmonics are numbered according to the ratio between their frequencies and that of the fundamental. See Bell (1981: 17–8).

7 Slutsky's model series were taken from NKFin (People's Commissariat of Finance) data obtained in drawing the numbers of a Soviet government lottery loan.

8 In the 1927 article Slutsky did mention Yule's (1926) article on nonsense correlations, and hence Yule's work might have been one of the inspirations for Slutsky's efforts in this respect.

9 George W. Bush should take note.

10 For a discussion of Tugan-Baranovsky's work on fluctuations, see Barnett (2001). For Pervushin, see Barnett (1996).

11 Frisch had contacted Slutsky personally when the idea of creating an econometric society was first proposed, and hence the connection between the two pioneers was direct.

12 Judy Klein has posited that the mental machinations of forming expectations could be a mechanism by which a moving summation of random disturbances was actually achieved in an economy. See Klein (1997: 278).

13 Differencing procedures are not always mentioned in this respect.

14 Marji Lines has suggested that from a methodological point of view, R.E. Lucas's business cycle theory followed more in the spirit of Slutsky than Frisch (Lines 1990: 359). Lines outlined a model in which random monetary shocks were filtered in the process of aggregate expectation formation so as to produce correlated price expectations, which led to the autocorrelated stochastic fluctuations known as business cycles (p. 369). However, Lines has fallen into the trap of attributing the idea of 'shocks' to Slutsky rather than to Frisch.

15 I am grateful to Professor Eugen Seneta for assistance in comprehending Slutsky's contribution on this particular topic. The stochastic limit should be distinguished from what is usually called Slutsky's theorem, which states that if X_n is a sequence of random variables that converges in probability to a, then a continuous function of X_n would converge in probability to a continuous function of a. See Davidson (1994: 286).

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Abstract

This article examines Slutsky's 1927 paper 'The Summation of Random Causes as the Source of Cyclic Processes'. It provides an in-depth analysis of both the content and the reception of Slutsky's groundbreaking contribution by distinguishing between a 'real' and a 'statistical' interpretation of Slutsky's two related hypotheses, and also discusses the context of composition of the paper in the Moscow Conjuncture Institute. It then places the 1927 paper in the context of Slutsky's other work in economics and statistics, and highlights some lines of influence that have emanated from it. Various latent ambiguities in Slutsky's ideas are considered.

Keywords

Business cycles, econometrics, statistics, Kuznets, Frisch